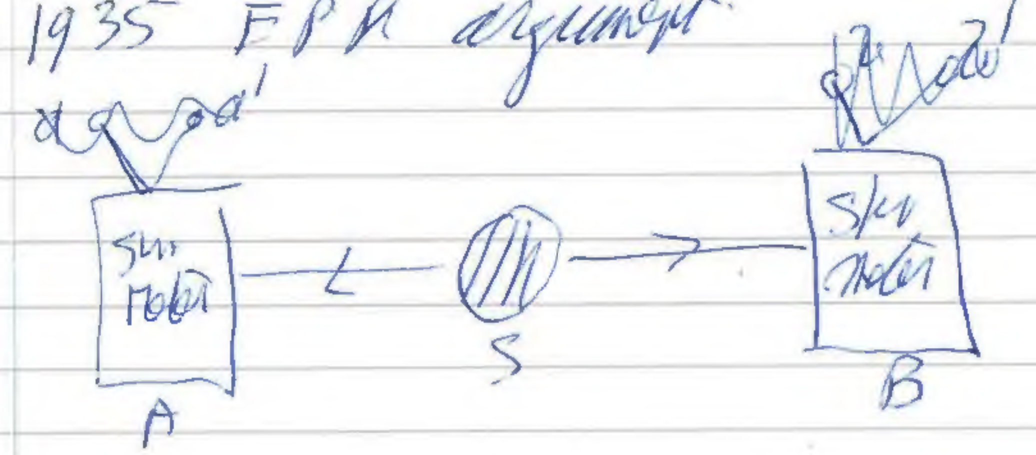
Algebraic Proofs of Nonlocality

Oxford February 1990

In what sense is QM a nonlocal theory? —
cf 2 slit experiment, extended wave functions
Efimov states in nuclear physics etc.

But RQFT is based on 'causality' axioms,
so how can the theory be nonlocal?

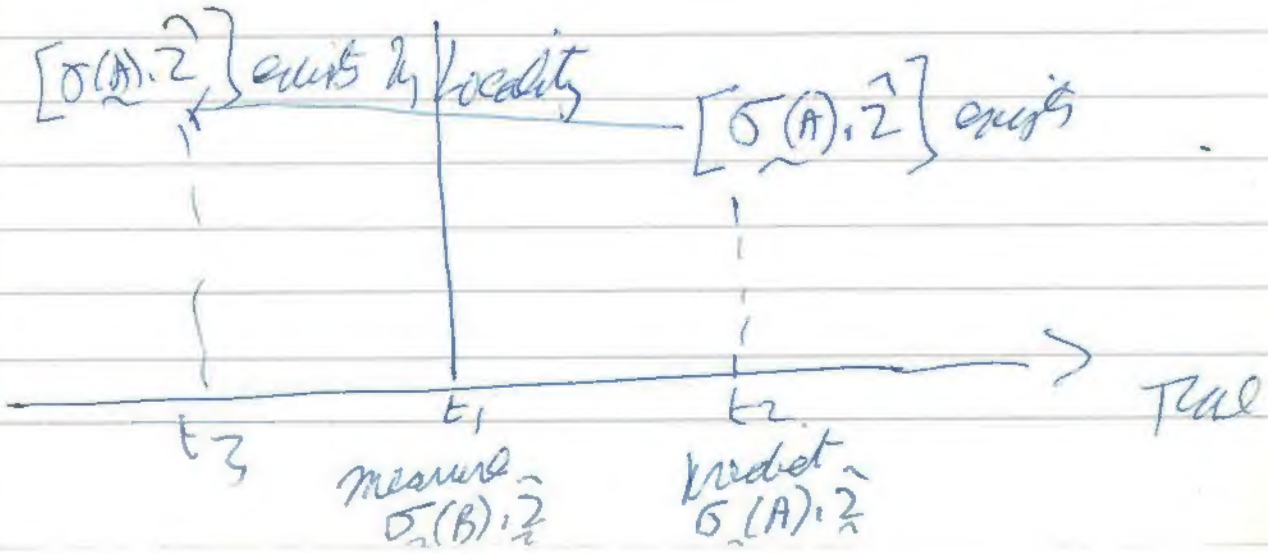
(1) 1935 EPR argument



Set $\hat{a} \parallel \hat{b} \parallel \hat{z}$ $\quad \Psi_{singlet} = \frac{1}{\sqrt{2}} \{ |\sigma(A)\cdot\hat{z} = +1\rangle$

$$|\sigma(B)\cdot\hat{z} = -1\rangle - |\sigma(A)\cdot\hat{z} = -1\rangle$$

$$|\sigma(B)\cdot\hat{z} = +1\rangle \}$$



(2)

So Einstein Dilemma

F → ~(L) v Incompleteness

on QM formalism ⟹ nonlocality or Incompleteness.

completed version of QM (hidden variables)

Bell Inequalities

violated by Expt.

nonlocality.

So QM is nonlocal simpliciter.

But Bohr denied the nonlocality was a "physical" effect.
Shd we reject Bohr's approach

then what about
? + Locality + hidden variables ⟹ Bell Ineq.

Early proofs Bell (1964) assume Determinism
+ probability structure (J.D. for incompatible observables). Also auxiliary assumptions ch. 6 - showing all

Two Controversies (1) Does proof of Bell, under determinism, commit one to J.D.?

(after Stapp 1971, Eberhard 1977) Fine says yes (1982)
Redhead says no (1983, 1988)
with Svetlichny, Butterfield, Barrett.

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Fine's 1982 theorem extended to beautiful mathematical theory by Pitowski and others — see Pitowski (1989) Quantum Probability – Quantum Logic

Generalized Bell inequalities are facet inequalities defining the facets of a multidimensional polytope.

(2) Does Can the Stapp-Eberhard proof be extended to indeterminism.

Stapp says yes

Hellman (1982) Redhead (Clifton, Butterfield) says no.
Redhead (1983, 1987).
CBR paper 1990.
A Stapp in the Wrong Direction

Can one give proofs of nonlocality in h.v. reconstructions that do not use probability theory?

Answer Assuming determinism yes (nearly)
Assuming indeterminism NO
But there do now exist ~~algebraic~~ so-called stochastic algebraic proofs of nonlocality (Elby, Clifton)
These proceed by starting with general stochastic framework and showing certain probs are 0 or 1 ~~then~~ then using these assignments to get an algebraic contradiction.

(4)

History of the algebraic proofs

1) Project: Derive a Kochen-Specker (1967) contradiction for two spin-½ systems. (cf Gleason (1957))

i.e. show local observables like $\underset{\sim}{\sigma}(A) \cdot \hat{a}$ must depend on a total context of properties of the whole system.

Proposed by Bub in 1976 in form of a question.
Cited Maczynski's theorem (1971) be- lieving that it be extendable from maximal to locally maximal observables. ?

But M's theorem was so extended by Demopoulos, Humphreys and Bub in 1980.

So no algebraic proof of nonseparability could be given.

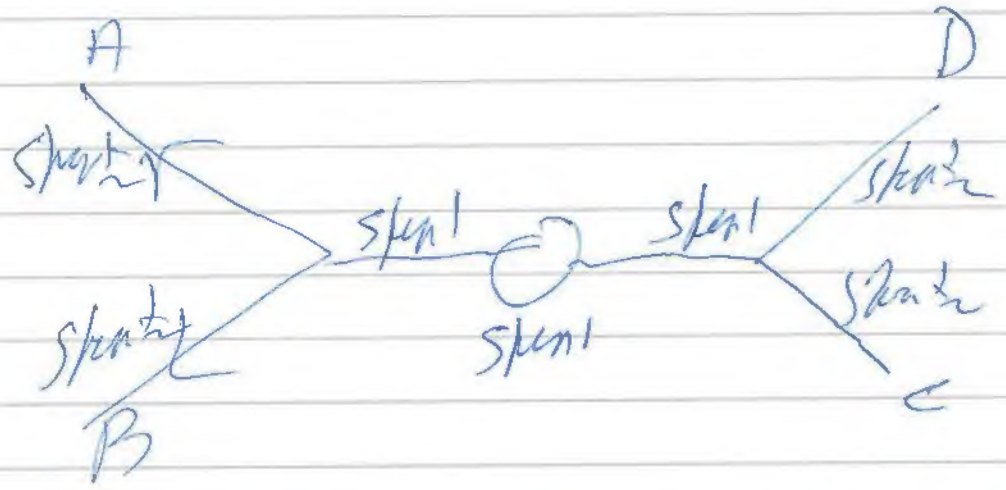
2) In 1983 Redhead's Desmond derived a K-S paradox on one of a pair of spin-one particles assuming separability and locality.
Involved locally non-maximal observables.

3) Stairs (1983) followed by Brown & Svetlichny (1990)
placed law to get a proof of nonlocality.
using similar assumption to R & H, but extended to locally maximal observables.
but again assuming determinism.

4) Early proposal of stochastic version of
Stapp – Brans – Svetlichny

5) Greenberger, Horne & Zeilinger (1989) produced a brand new "algebraic" proof quite unrelated to K–S & Gleason.

6) 1990 Redhead & Clifton showed the stated proof given by Greenberger could not be made to work but Clifton devised a modified proof that does work.



Then it is shown that and if $\theta_A+\theta_B-\theta_C-\theta_D=\pi$, $A(\theta_A)B(\theta_B)C(\theta_C)D(\theta_D)=+1$

if $\theta_A+\theta_B-\theta_C-\theta_D = 0$,

$$A(\theta_A)\,B(\theta_B)\,C(\theta_C)\,D(\theta_D) = -1$$

Consider ~~4~~ 5 possible settings for $\{\theta_A\ \theta_B\ \theta_C\ \theta_D\}$

	(1)	(2)	(3)	(4)	~~ABCD~~ (5)
θ_A	$\rightarrow$	$\downarrow$	$\rightarrow$	$\uparrow$	$\{\downarrow\}$
θ_B	$\rightarrow$	$\uparrow$	$\uparrow$	$\rightarrow$	$\rightarrow$ / $\uparrow$
θ_C	$\rightarrow$	$\rightarrow$	$\uparrow$	$\uparrow$	
θ_D	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$
	$ABCD = -1$	$ABCD = -1$	$ABCD = -1$	$ABCD = -1$	$ABCD = +1$

From (4) & (5) $A(\uparrow) = -A(\downarrow)$

Hence we obtain

(1) × (2) $A(\rightarrow)\, B(\rightarrow) = A(\downarrow)\, B(\uparrow)$

(1) × (3) similarly $B(\rightarrow)\, C(\rightarrow) = B(\uparrow)\, C(\uparrow)$

(1) × (4) and $A(\rightarrow)\, C(\rightarrow) = A(\uparrow)\, C(\uparrow)$

Multiplying $1 = A(\downarrow)\, A(\uparrow)$

or $A(\downarrow) = A(\uparrow)$, But $A(\downarrow) = -A(\uparrow)$ from (4) & (5)

∴ contradiction

Greenberger considers this argument a counterargument to Bohr's response to EPR. He talks of restricting the discussion to the "superclassical case", "where an element of reality exists by virtue of perfect predictability"

But this is running the 8 quantities

$A(\rightarrow), A(\uparrow) A(\downarrow), B(\rightarrow), B(\uparrow), C(\rightarrow), C(\uparrow), D(\rightarrow)$

involved in the argument do not all exist in the same experimental context, so counterfactual formulations of locality are required, and thus struggles in definiteness.

7) Clifton has generalized the GHZ argument to the stochastic case, in a manner parallel to what Eberhard did for Stairs

So where are we left?

correlations that cannot be explained

not causal

cf Polkinghorne's account of Redhead (1986) and no-signalling theorems of Ghirardi Rimini, Weber (1980) and others (including Eberhard, Page, Shimony, Redhead etc).

Recent work on violations of Bell inequalities in R QFT derived by eg. Summers, Werner, Landau, Fredenhagen etc. - adapt the proof

See also p.115 of my book.